LINEAR CIRCUIT ANALYSIS (EED) – U.E.T. TAXILA | 02

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INTRODUCTION

To actually determine the values of voltage or current of elements in given circuit, requires study of basic laws.

The basic laws: Ohm’s law and Kirchhoff’s laws, form the foundation of electric circuit analysis.

In addition to these laws, some techniques are applied for circuit design and analysis.

These techniques includes combining resistors in series or parallel, voltage division, current division, and delta-to-wye and wye-to-delta transformations.

OHM'S LAW

Ohm's Law states that the voltage across a resistor is directly proportional to the current flowing through the resistor.

Mathematically;

$$v \propto i$$

$$\boxed{v = iR}$$

Where 'R' is known as resistance of element (Resistor).

RESISTANCE

Materials in general have a characteristic behavior of resisting the flow of electric charge.

This physical property, or ability to resist current, is known as resistance.

The Resistance of an element denotes its ability to resist the flow of electric current, measured in ohms (Ω).

Mathematically;

$$R = \frac{v}{i}$$

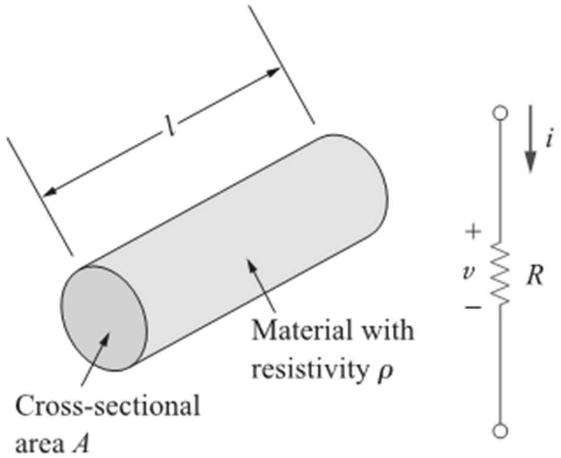
$$1\ \Omega = 1\ \mathrm{V/A}$$

RESISTANCE

The resistance of any material depends on cross-sectional area A and length l.

Mathematically;

$$R = \rho \frac{\ell}{A}$$



Where ρ is resistivity of material, measured in Ωm.

RESISTIVITY

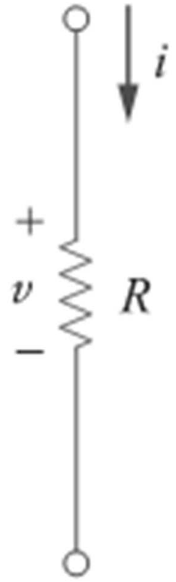
Resistivities of common materials

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

PASSIVE SIGN CONVENTION

To apply Ohm's law on resistor, the direction of current i and the polarity of voltage v must conform the passive sign convention.

The Passive Sign Convention implies that current flows from a higher potential to a lower potential, $v=iR$. If the current flows from a lower potential to higher potential, $v=-iR$.



RESISTOR

The circuit element used to model the current-resisting behavior of a material, is known as Resistor.

Resistor is passive element.

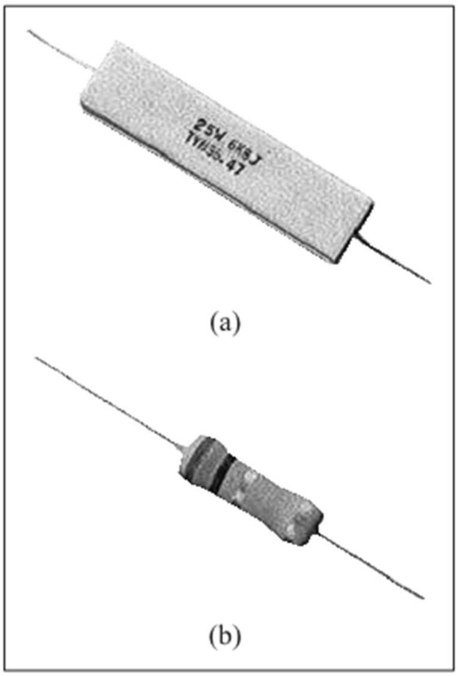
There are two major types of resistors: fixed resistor and variable resistor.

FIXED RESISTOR

Fixed Resistor is that which exhibits the constant value of resistance.

There are two common types of fixed resistors: wire wound and composition.

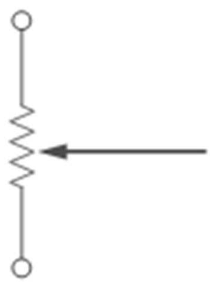
The wire wound resistors are used for large values of resistance.



VARIABLE RESISTOR

Variable Resistor is that which exhibits adjustable value of resistance.

The variable resistor is also known as Potentiometer.

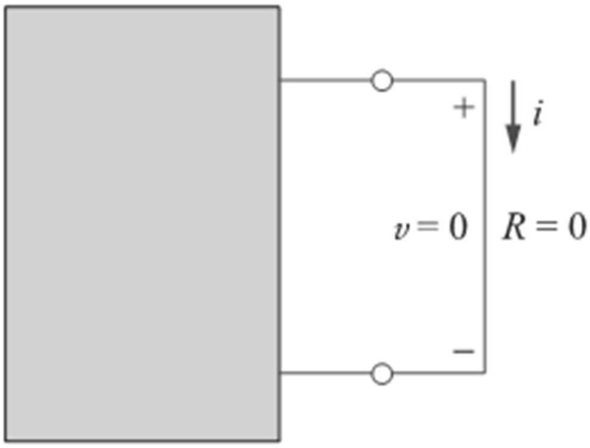


VALUE OF RESISTOR

Since value of R can range from zero to infinity, it is important to consider extreme possible values of R.

A Short Circuit is a circuit element with resistance approaching to zero.

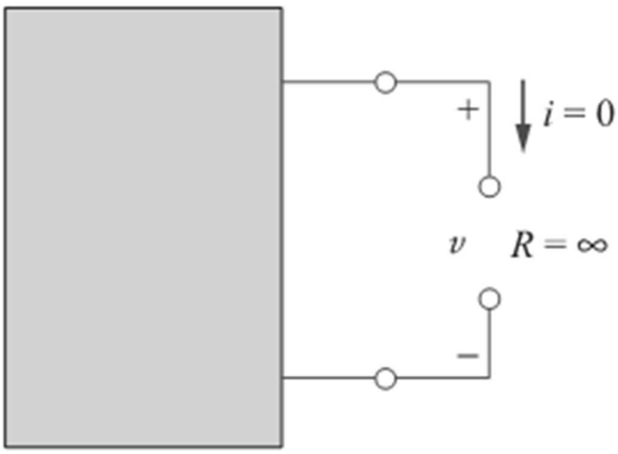
$$v = iR = 0$$



VALUE OF RESISTOR

An Open Circuit is a circuit element with resistance approaching to infinity.

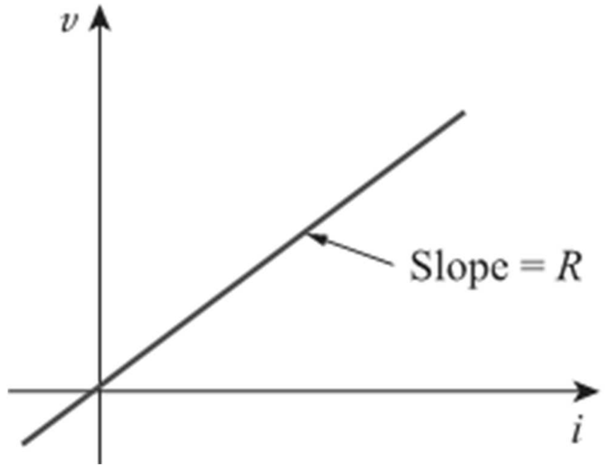
$$i = \lim_{R \to \infty} \frac{v}{R} = 0$$



LINEAR RESISTOR

A resistor that obeys Ohm's law is known as Linear Resistor.

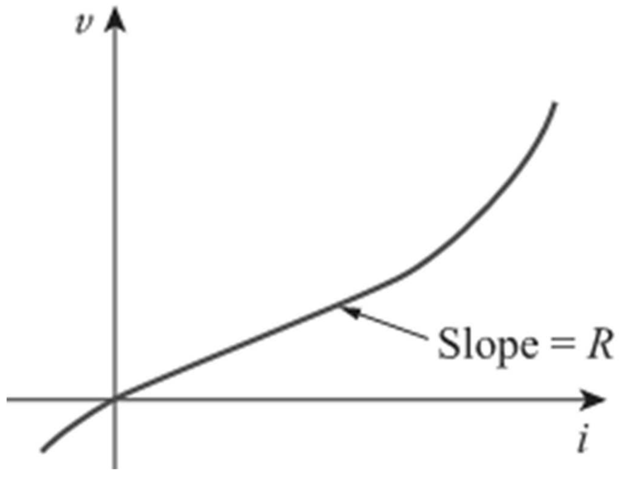
Its has a constant resistance and thus its current-voltage characteristic (i-v graph) is straight line passing through origin.



NON-LINEAR RESISTOR

A resistor that does not obey Ohm's law is known as Non-Linear Resistor.

Its resistance varies with current its current-voltage characteristic (i-v graph) is not straight line.



CONDUCTANCE

Conductance is the ability of an element to conduct electric current; measured in mhos (℧) or siemens (S).

$$G = \frac{1}{R} = \frac{i}{v}$$

$$1\ \mathrm{S} = 1\ \mho = 1\ \mathrm{A/V}$$

$$i = Gv$$

Conductance is the reciprocal of resistance and is a useful quantity in electric circuit analysis.

POWER DISSIPATION IN RESISTOR

The power dissipated by a resistor may be expressed in terms of resistance as well as conductance.

$$p = vi = i^2R = \frac{v^2}{R}$$

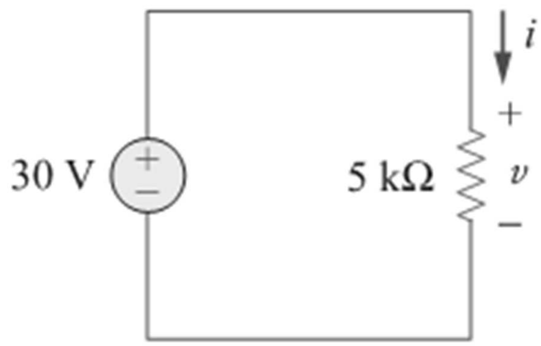
$$p = vi = v^2G = \frac{i^2}{G}$$

PROBLEMS

An electric iron draws 2 A at 120 V. Find Resistance?

(60 Ω)

In circuit, calculate current, conductance and power?

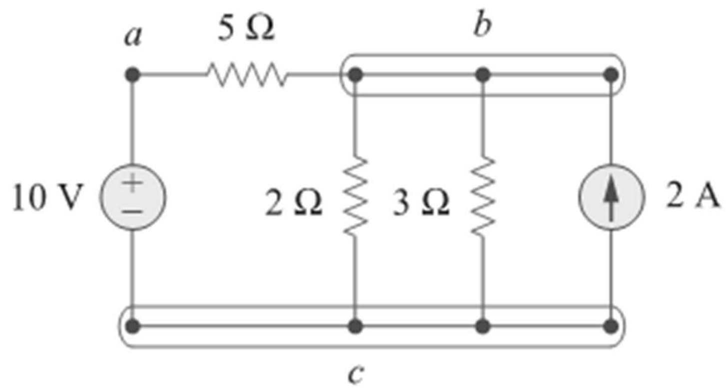


(6 mA, 0.2 mS, 180 mW)

BRANCH

A Branch represents a single element such as a voltage source, current source or a resistor.

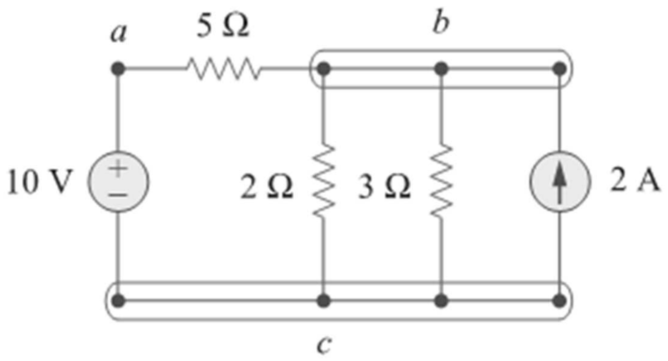
In other words, a branch represents any two-terminal element.

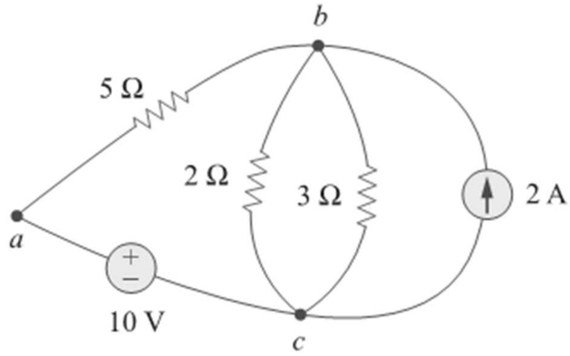


NODE

A Node is the point of connection between two or more branches.

If a short circuit connects two nodes, the two nodes constitute a single node.



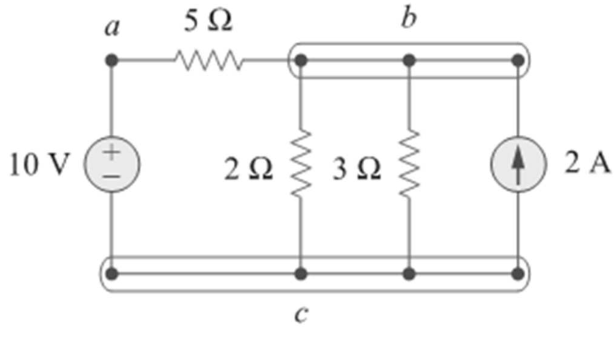


LOOP

A Loop is any closed path in a circuit.

A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.

A loop is said to be independent if it contains at least one branch which is not a part of any other loop.



LOOP

The loop $abca$ with 2 Ω resistor and loop $abca$ with 3 Ω resistor are examples of independent loops.

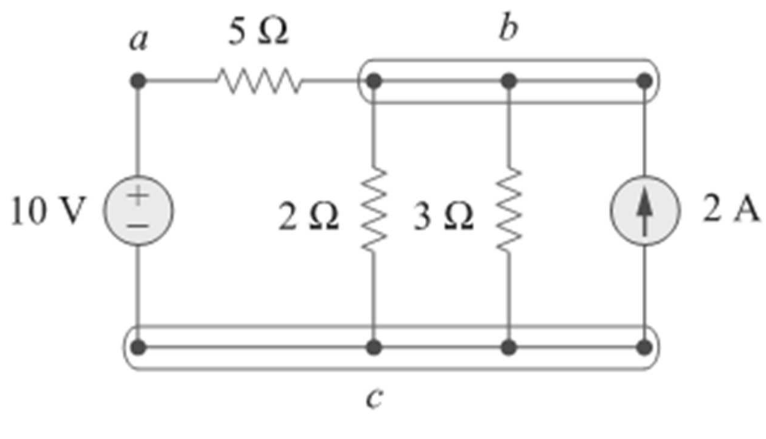
A network with b branches, n nodes and l independent loops will satisfy the fundamental theorem of network topology.

$$b = l + n - 1$$

SERIES CONNECTED ELEMENTS

Two or more elements are in Series if they exclusively share a single node and consequently carry the same current.

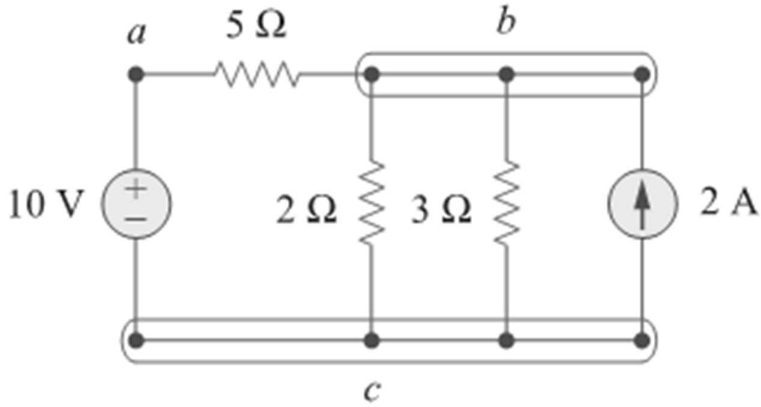
Elements are in series when they are chain-connected.



PARALLEL CONNECTED ELEMENTS

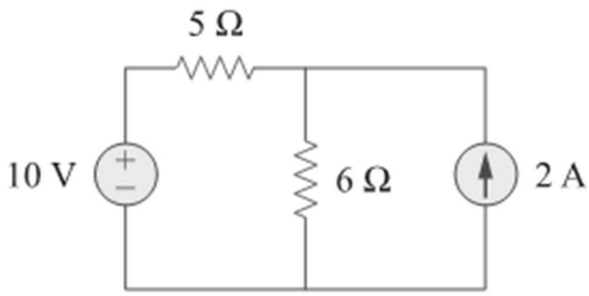
Two or more elements are in Parallel if they are connected to same two nodes and consequently have the same voltage across them.

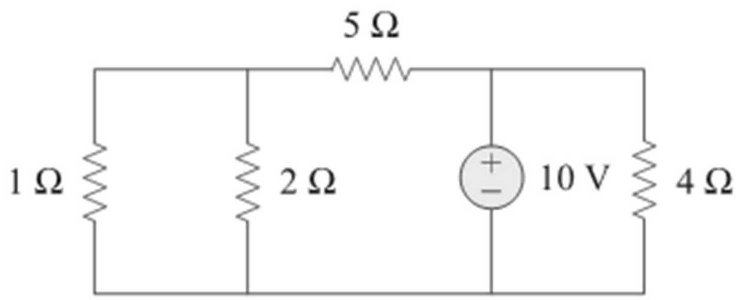
Elements are in parallel when they are connected to same pair of nodes.



PROBLEMS

Determine number of branches and nodes in circuits? Also identify which elements are connected in series and parallel?





KIRCHHOFF'S CURRENT LAW (KCL)

Kirchhoff's Current Law (KCL) states that the algebraic sum of currents entering a node is zero.

Mathematically;

$$\sum_{n=1}^{N} i_n = 0$$

The KCL is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

To prove KCL, assume set of currents into a node.

$$i_T(t) = i_1(t) + i_2(t) + i_3(t) + \cdots$$

KIRCHHOFF'S CURRENT LAW (KCL)

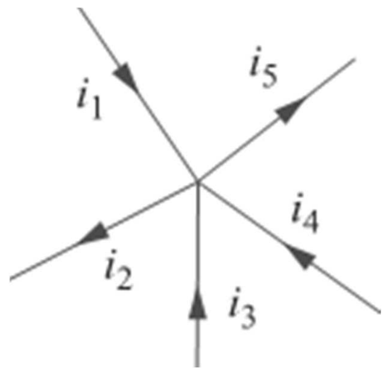
Integrating both sides;

$$q_T(t) = q_1(t) + q_2(t) + q_3(t) + \cdots$$

By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

$$i_1 + i_3 + i_4 = i_2 + i_5$$

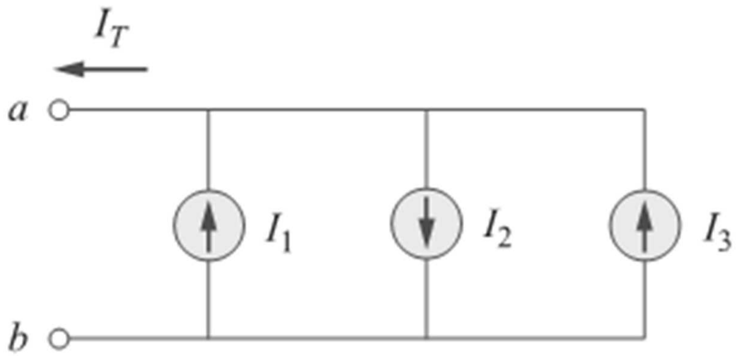


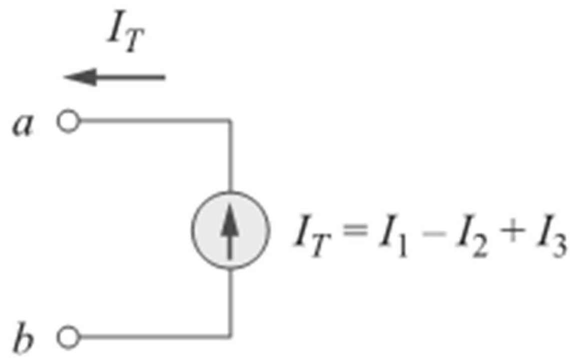
KIRCHHOFF'S CURRENT LAW (KCL)

The sum of currents entering a node is equal to the sum of current leaving the node.

A simple application of KCL is combining the current sources in parallel.

$I_T + I_2 = I_1 + I_3$ $\qquad$ $I_T = I_1 - I_2 + I_3$





KIRCHHOFF'S VOLTAGE LAW (KVL)

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of all voltages around a closed path (loop) is zero.

Mathematically;

$$\sum_{m=1}^{M} v_m = 0$$

KVL is based on the law of conservation of energy.

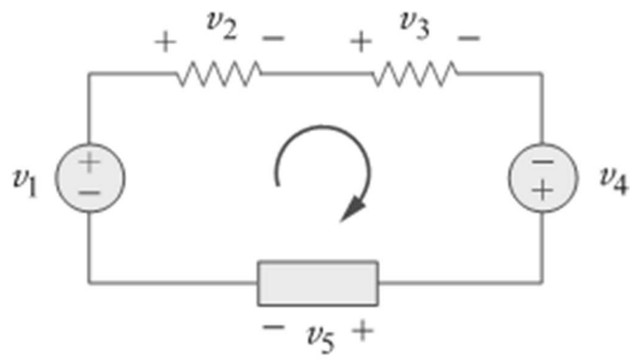
KVL can be applied by taking clockwise or counter-clockwise trip around the loop.

KIRCHHOFF’S VOLTAGE LAW (KVL)

The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop.

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_2 + v_3 + v_5 = v_1 + v_4$$



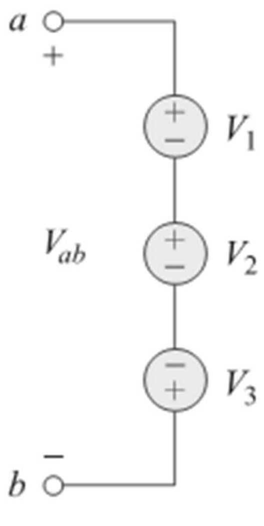
Sum of voltage drops = Sum of voltage rises

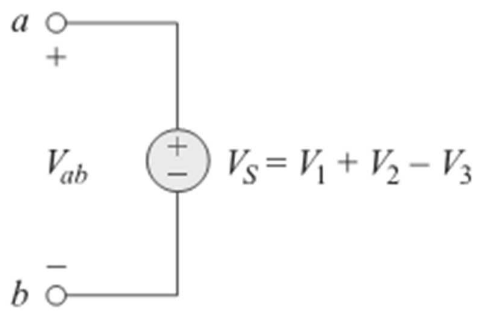
KIRCHHOFF’S VOLTAGE LAW (KVL)

The application of KVL is combining the voltage sources.

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

$$V_{ab} = V_1 + V_2 - V_3$$

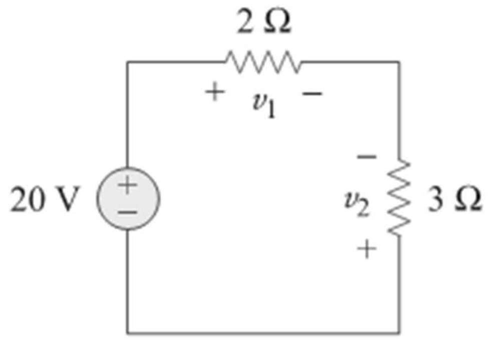




PROBLEMS

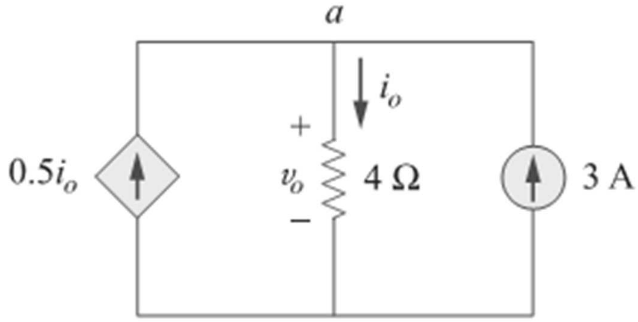
Find the voltages v_1 and v_2?

(8 V, -12 V)



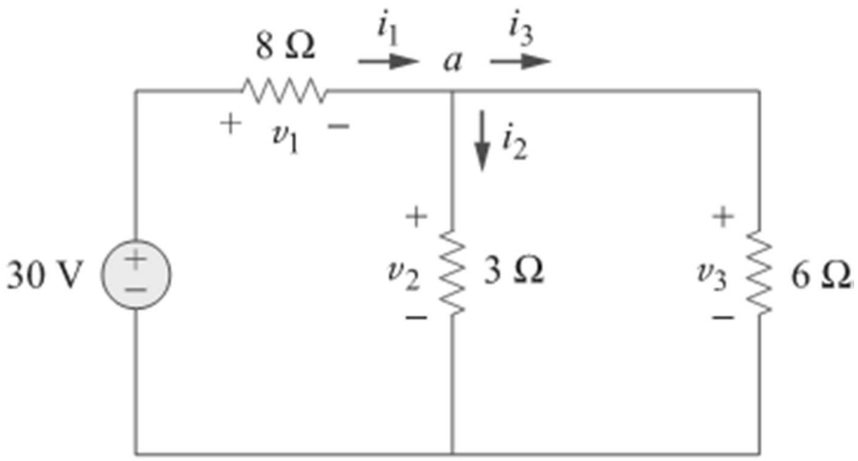
Find the current i_o and voltage v_o?

(6 A, 24 V)



PROBLEMS

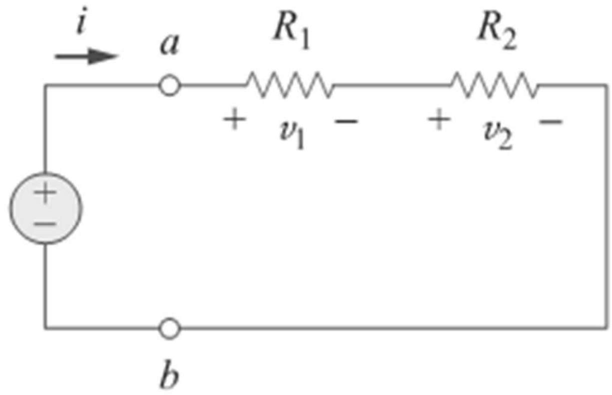
Find currents and voltages in the circuit?



(3 A, 2 A, 1 A, 24 V, 6 V, 6 V)

SERIES RESISTORS AND VOLTAGE DIVISION

Consider the series circuit;



Applying Ohm's law;

$$v_1 = iR_1, \qquad v_2 = iR_2$$

Applying KVL;

$$-v + v_1 + v_2 = 0$$

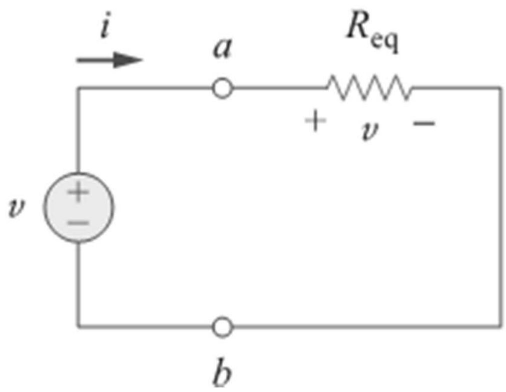
$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$i = \frac{v}{R_1 + R_2} \qquad v = iR_{eq}$$

$$R_{eq} = R_1 + R_2$$

SERIES RESISTORS AND VOLTAGE DIVISION

Voltages across each resistor;



$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

The equivalent resistance of any number of resistors connected in Series is the sum of their individual resistances.

SERIES RESISTORS AND VOLTAGE DIVISION

If there are N resistors connected in series;

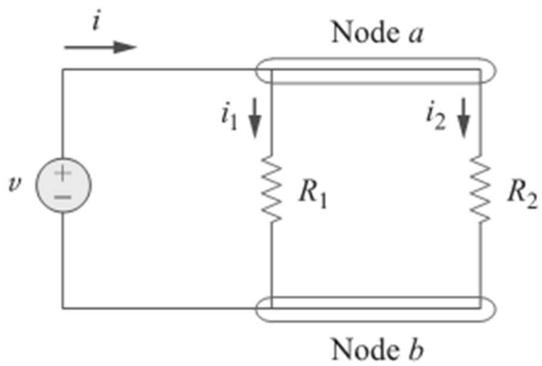
$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

The principle of voltage division for N resistors is expressed as;

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

PARALLEL RESISTORS AND CURRENT DIVISION

Consider the parallel circuit;



Applying Ohm's law;

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

Applying KCL;

$$i = i_1 + i_2$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

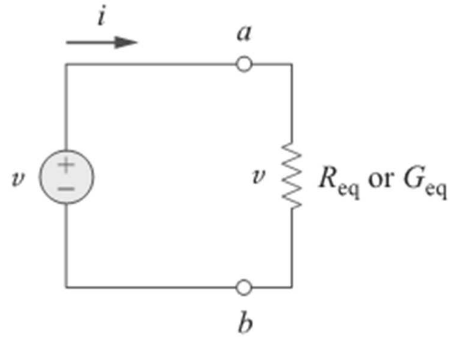
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

PARALLEL RESISTORS AND CURRENT DIVISION

Current through each resistor;

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$



The equivalent resistance of two Parallel resistors is equal to the product of their resistances divided by their sum.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

PARALLEL RESISTORS AND CURRENT DIVISION

The equivalent conductance of resistors connected in Parallel is the sum of their individual conductances.

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

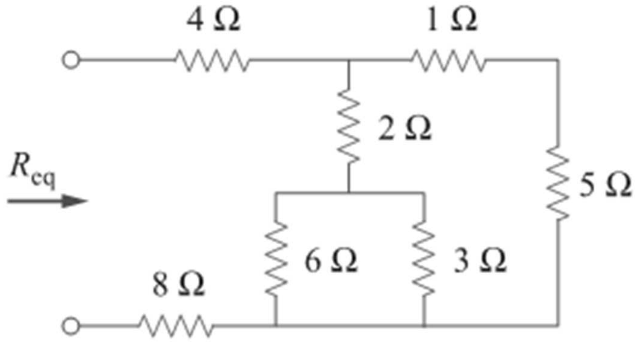
The principle of current division for N resistors is expressed as;

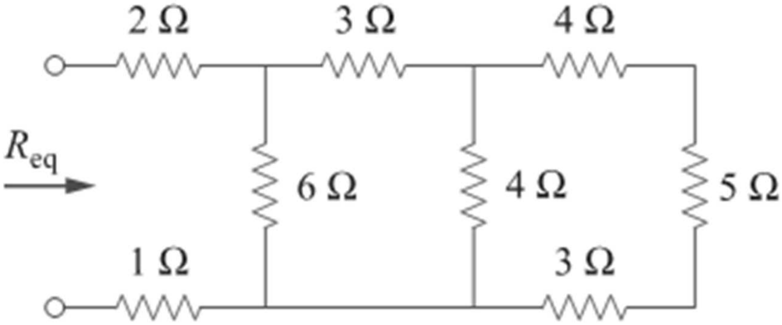
$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i$$

PROBLEMS

Find equivalent resistance?

(14.4 Ω)





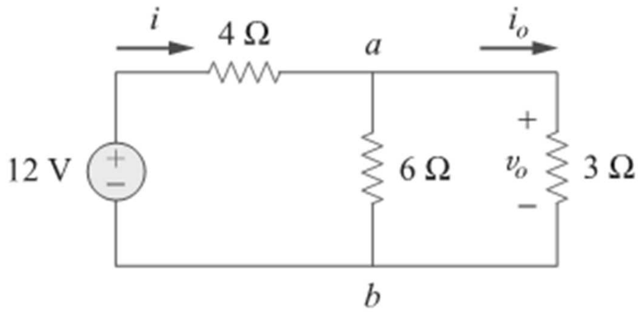
(6 Ω)

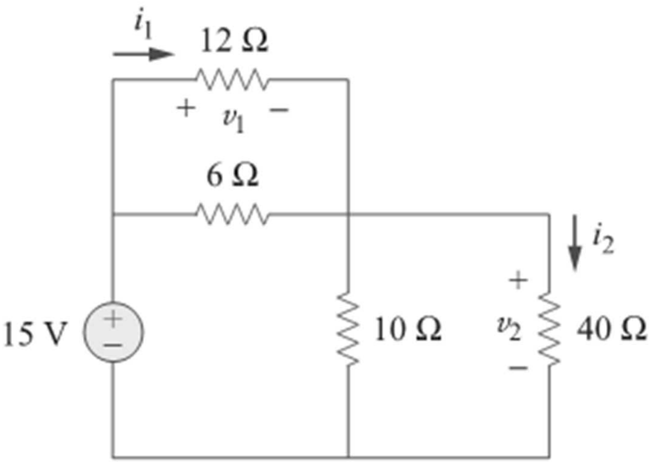
PROBLEMS

Find voltages and

currents?

(4 V, 4/3 A)





(5 V, 10 V,

416.7 mA, 250 mA)

WYE AND DELTA CONNECTED RESISTORS

Sometimes, the resistors are neither in parallel connected nor in series.

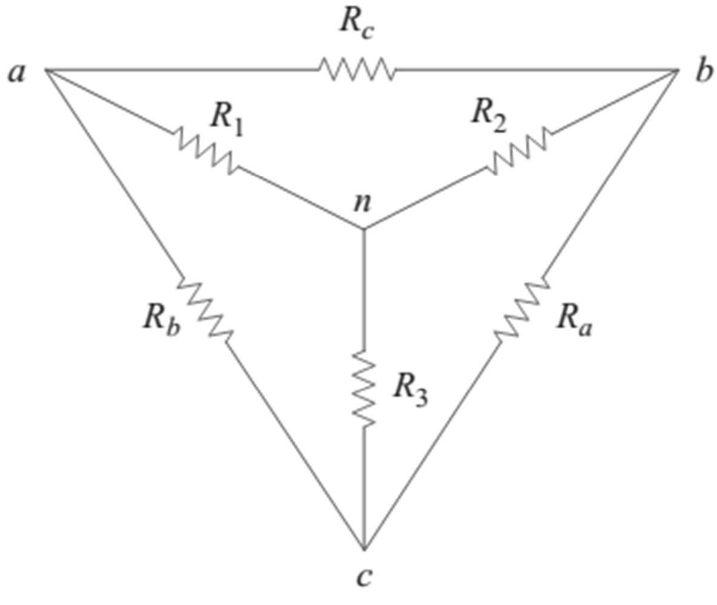
Such complex pattern of resistors is solved by considering them in wye or delta.

DELTA-WYE TRANSFORMATION

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

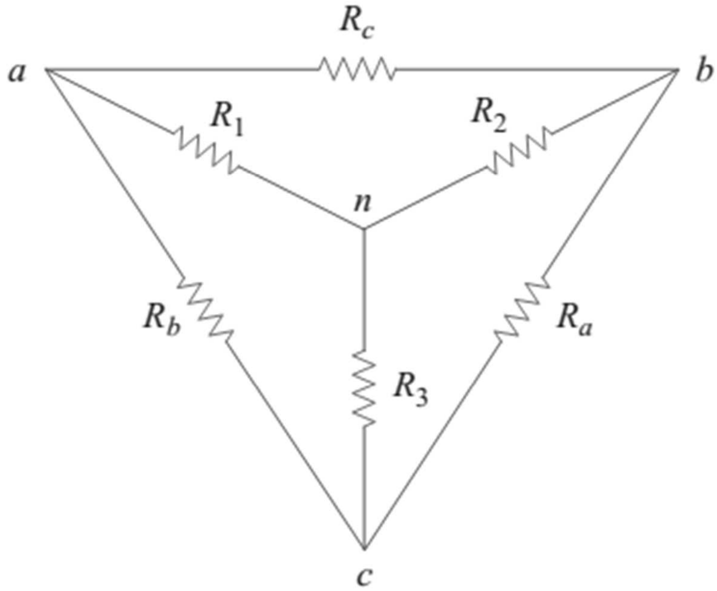


WYE-DELTA TRANSFORMATION

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

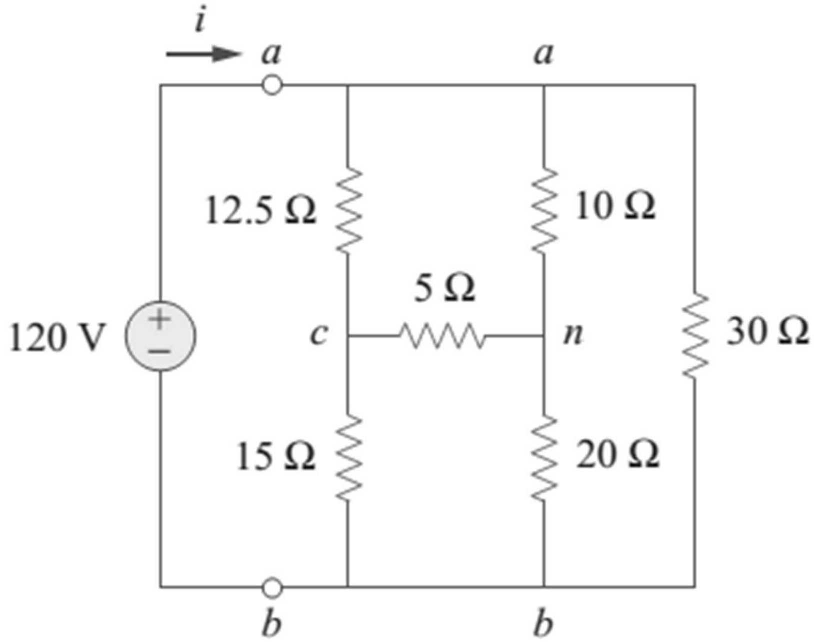
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$



PROBLEMS

Find equivalent resistance?



(9.632 ohm)

REFERENCES

Fundamentals of Electric Circuits (4th Edition)

Charles K. Alexander, Matthew N. O. Sadiku

Chapter 02 – Basic Laws (2.1 – 2.7)

Exercise Problems: 2.1 – 2.75

Do exercise problem yourself.